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Variable Node Drag Parameterization for Re-entry Trajectory Estimation

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A numerical parameter optimization technique is used to determine the piecewise continuous polynomial approximation to the drag coefficient history that produces a re-entry trajectory that best matches radar data in a weighted least-squares sense. The drag coefficient is parameterized in terms of node point locations, and polynomial coefficients or function values at the nodes. Estimation of node locations for drag coefficient histories using simulated radar data has reduced data residuals by almost one-half through improved approximation of the sharp rise in drag coefficient caused by boundary layer transition.

Nomenclature

a	= parameters which determine node spacing
C_D	= drag coefficient
f	= vector of first derivatives of state equations
F	= function for data fitting example
g	= function relating states to observations
J	= quadratic form to be minimized
m	= total number of parameters to be estimated
N	= number of observations
n	= number of points at which function is estimated
t	= time
W	= weighting matrix
x	= state variables
y	= observed variables
ϵ	= error between observed and estimated variables
$(\cdot)$	= differentiation with respect to time

Introduction

A STANDARD technique for solving optimal control[1,2] or estimation problems[3] is to parameterize the control or any unknown functions associated with the estimation problem and reduce the original problem to a parameter optimization problem. The solutions obtained are then to some extent dependent on the choice of the parameterization. If the function to be determined has rapid changes over short intervals, then parameterization using splines or piecewise continuous polynomials often cannot accurately represent the true function.[4] This would be true, for example, for an optimal control problem where the optimal control switches directly from one boundary to another (bang-bang control problem). It is also true for the problem of estimating the trajectory and drag coefficient history for a re-entry vehicle using radar data. When the boundary layer on the vehicle changes from laminar to turbulent, the drag coefficient history experiences a sharp rise. Without a priori knowledge of the transition time interval, it is difficult to accurately model this portion of the trajectory with standard piecewise continuous polynomials. The nodes of the polynomial, in the absence of a priori knowledge of the true answer, may be incorrectly located and it will be difficult for the polynomials to produce the rapid rise in the drag coefficient history associated with the proper transition time.

The approach used to solve the trajectory estimation problem described in this paper approximates the drag coefficient using piecewise continuous functions. A variable metric parameter optimization technique[5] is used to find the approximation to the drag coefficient history which best matches the radar data in a weighted least-squares sense. This approach also allows the nodes of the function to be treated as parameters. The optimization program then varies the node location simultaneously with the function coefficients or drag coefficient magnitude at the nodes. Thus the program can automatially locate the proper transition initiation and termination times to best fit the radar data.

Problem Statement

The problem of estimating a vehicle's trajectory and drag coefficient history from radar data will be briefly described below. If a point mass model of the dynamical motion is used, then the differential equations describing the motion of the vehicle may be written as

$$\dot{x}=f(x,C_D) \tag{1}$$

where x is a six-vector consisting of three position and three velocity components. The drag coefficient is an unknown function of time which is to be estimated from radar data. The state-observation model can be written as

$$y_i=g(x_i)+\epsilon_i \qquad (i=1,2,...,N) \tag{2}$$

where y is a three-vector consisting of range, azimuth, and elevation measurements at discrete time intervals, t_i. The estimation problem is then to determine the initial conditions for the state equations (1), x_0, along with the drag coefficient history, $C_D(t)$, which causes the predicted radar measurements from the mathematical model to fit the actual data in some best sense. If a weighted least-squares solution is to be computed, then x_0 and C_D are computed, which causes

$$J=\frac{1}{2}\sum_{i=1}^{N}\epsilon_i^T W\epsilon_i \tag{3}$$

to be minimum.

This approach has been used at Sandia to compute trajectories and drag coefficient histories for the flight test data obtained during a large number of re-entry vehicle flight tests. These results were obtained by assuming that the drag coefficient was either a straight line or piecewise continuous polynomial. The estimation program then adjusted both the initial conditions and values of the drag coefficient history at the nodes to produce a solution. While most of the results obtained in this manner have been very good, the drag coefficient history in some instances does not seem to properly model some expected rapid changes in C_D.

When the boundary layer on the vehicle changes from laminar to turbulent, for example, the drag coefficient for a

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small re-entry vehicle may increase 10-15%. It is almost impossible to observe where this change starts and stops from direct examination of radar data. Thus it is not possible to select beforehand the node locations to coincide with these times. If this is not done, and a very large number of node locations is not used, then a rapid change in drag coefficient is smeared over several node points. This inability to properly model the drag coefficient during transition also causes the trajectory to be incorrect over other time intervals since the velocity after integrating the mathematical model through transition will not be correct. Thus a scheme which can more closely identify the change in the drag coefficient during transition will not only improve the estimate of C_D but also the estimate of the trajectory over the entire interval.

A similar phenomenon occurs for small re-entry vehicles when the boundary layer at the nosetip becomes turbulent. This results in a significant nosetip shape change which also causes significant changes in C_D which cannot be seen directly from radar data. Again, some scheme is needed which provides more freedom in allowing the assumed piecewise continuous polynomials to accurately represent rapid changes in C_D caused by nosetip shaping.

One way of allowing the assumed functional form for C_D to more closely approximate a function, which may have large rapid changes, is to allow the node points to move.[6-8] Thus the node location, as well as the value of C_D at the nodes, could be treated as parameters. This allows the program the flexibility of moving nodes to the start and end times of the rapid changes and provides much more flexibility in the assumed piecewise continuous polynomial approximation. This idea is briefly outlined below.

When the nodes are fixed, a straight-line approximation for C_D is

$$C_D = C_{D_i} + b(t - t_i) \quad (t_i \le t \le t_{i+1})$$
$$b = (C_{D_{i+1}} - C_{D_i}) / (t_{i+1} - t_i) \tag{4}$$

where the C_{D_i} are the values of C_D at t_i and are treated as the parameters with t_i fixed. If the node points are also to be treated as parameters, then this is easily done by adding the additional parameters, a_j, where

$$t_i = t_f \Big/ \left(1 + \sum_{k=i}^{n-2} a_{n-2-k}^2\right) \tag{5}$$

Note that t_0 and t_f are assumed fixed. Thus, if there are three intermediate times and $t_0 = 0$,

$$t_3 = t_f / (1 + a_0^2) \tag{6}$$
$$t_1 = t_f / (1 + a_0^2 + a_1^2 + a_2^2)$$

This choice of the a's as parameters keeps the values of t_i bounded by t_f and t_0, while also keeping them properly ordered so that

$$0 \le t_1 \le t_2 \le t_3 \le t_f \tag{7}$$

This formulation is readily adapted to nonlinear parameter optimization programs which can be used to find x_0, C_{D_i}, and a_j in order to produce a solution which causes the measure of error, J, to be a minimum. This approach with fixed nodes is described in Ref. 3. Numerical results for a re-entry problem and a simple data fitting problem which demonstrates the ideas discussed above are contained in the next sections.

Example Problem

The basic concept described above is that piecewise continuous polynomials with fixed preselected nodes often cannot properly model functions which may have large rapid changes in their values. This idea is illustrated by attempting

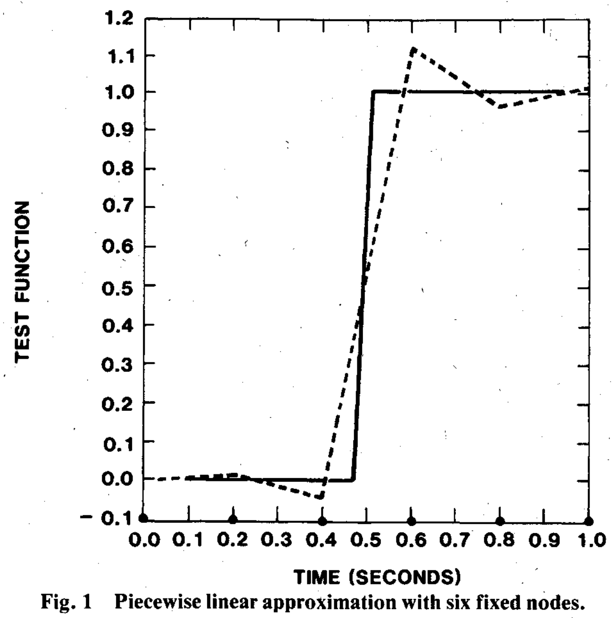

Fig. 1 Piecewise linear approximation with six fixed nodes.

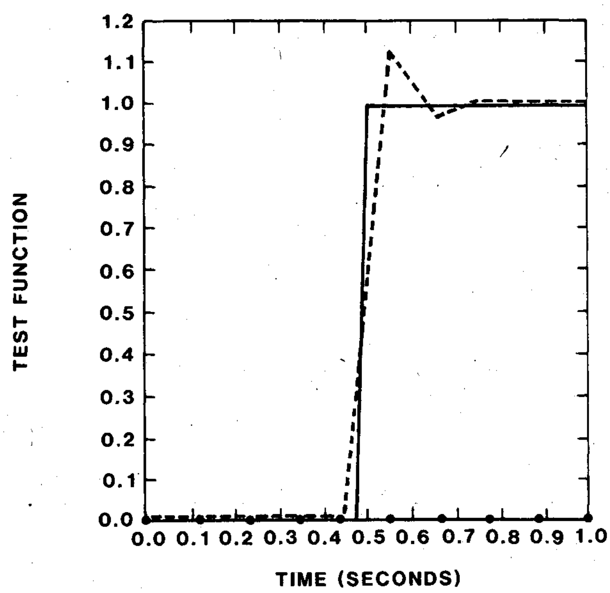

Fig. 2 Piecewise linear approximation with ten fixed nodes.

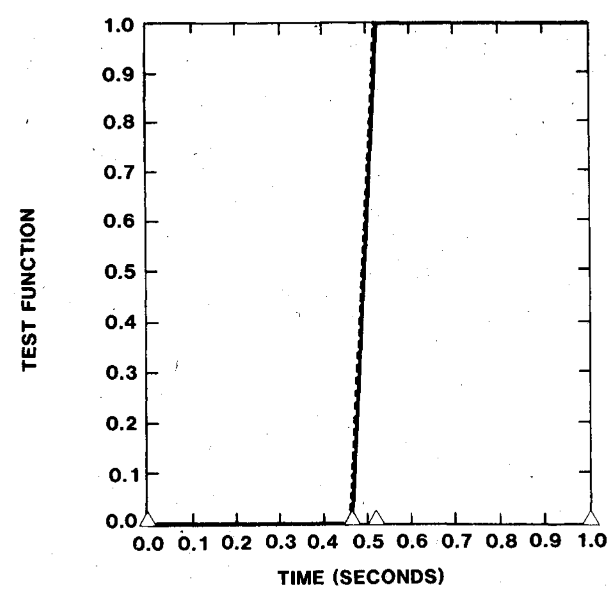

Fig. 3 Piecewise linear approximation with two variable nodes.

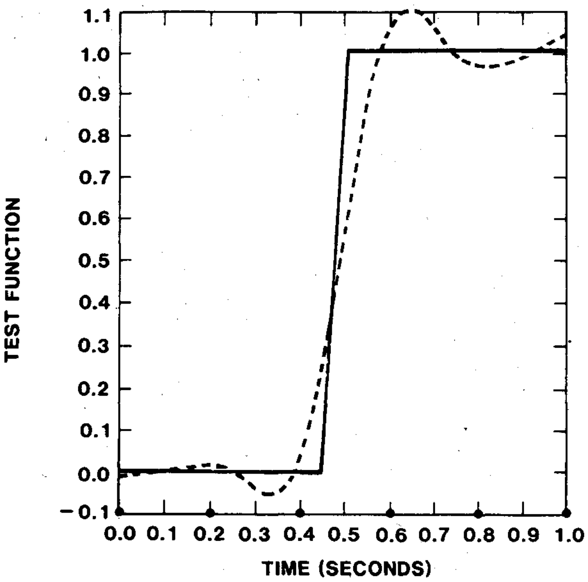

Fig. 4 Piecewise cubic approximation with six fixed nodes.

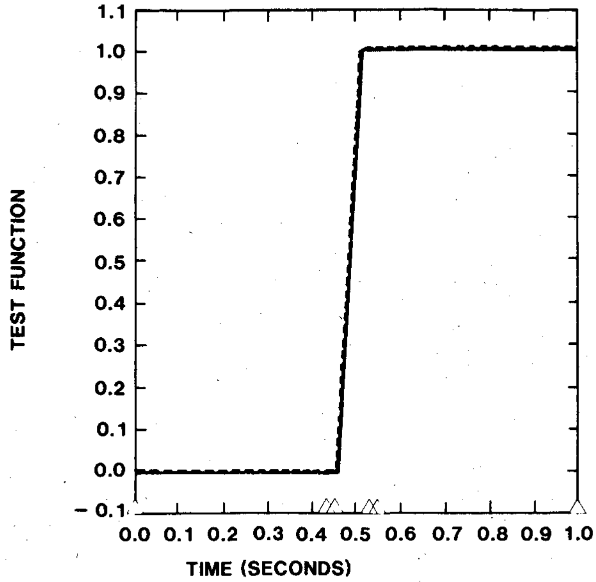

Fig. 6 Piecewise cubic approximation with four variable nodes.

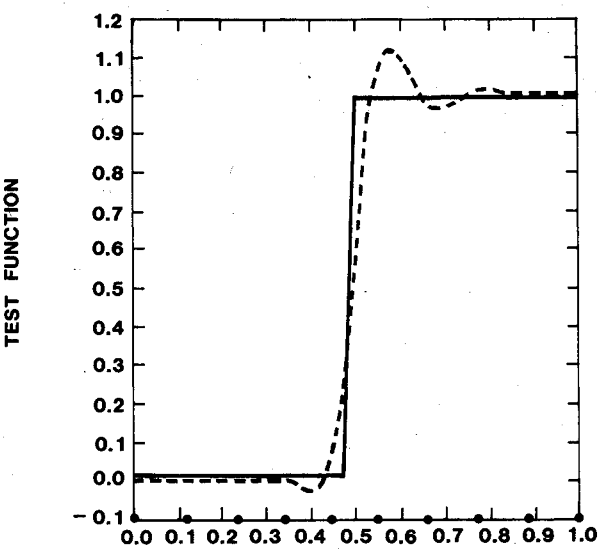

Fig. 5 Piecewise cubic approximation with ten fixed nodes.

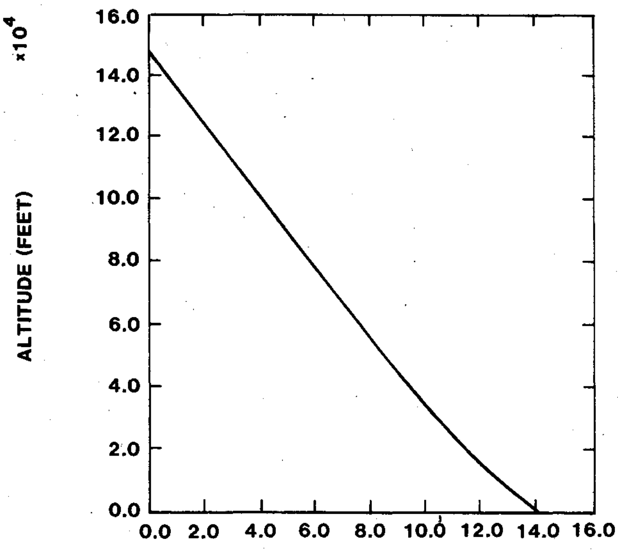

Fig. 7 Simulated re-entry altitude history.

to produce a piecewise continuous polynomial approximation to a function which has a ramped change at some assumed initially unknown time. This is essentially the same problem as attempting to approximate the expected ramp change in drag coefficient for a re-entry vehicle as the boundary layer becomes turbulent.

Consider the problem of approximating the function $F(t)$, given by

$$
\begin{aligned}
F(t) &= 0 && 0 \le t < 0.463 \\
F(t) &= 200(t-0.463) && 0.463 \le t \le 0.513 \qquad (8)\\
F(t) &= 1.0 && t > 0.513
\end{aligned}
$$

The approximating function can be written in terms of n function values at n selected node points, t_i ($i=1,2,3,\ldots,n$) and a specified interpolation procedure. The approximation of $F(t)$ with fixed uniformly distributed nodes is shown as the dashed line in Fig. 1 ($n=6$) and Fig. 2 ($n=10$). Linear interpolation was used over the uniform node distribution to obtain the value of the approximating function at the observed data points to form the residuals, ϵ. This has the effect of using piecewise continuous linear functions as the approximation technique. Node locations are indicated by the circular symbols on the abscissa.

The approximation of $F(t)$ can be improved by using a nonuniform distribution of node points. The least-squares approximation consists of determining the n values of the function and the $n-2$ values of a_j (a total of $m=2n-2$ parameters). The approximation obtained using four node points ($n=4$, $m=6$) is shown in Fig. 3. The nodes were uniformly distributed to start the numerical function minimization process and their converged locations are indicated by the triangular symbols. These locations are essentially the same as the break points of the example function. This is expected since only four node points are required to exactly linearly interpolate the given function.

The same function was approximated using cubic interpolation with continuity through first derivative. The results of this approximation are shown in Fig. 4 ($n=6$, $m=6$, nodes fixed), Fig. 5 ($n=10$, $m=10$, nodes fixed), and Fig. 6 ($n=6$, $m=10$, nodes free). Note that in Fig. 6 the four free interior nodes have converged to form two pairs arranged on either side of the ramp to accommodate the rapid change in

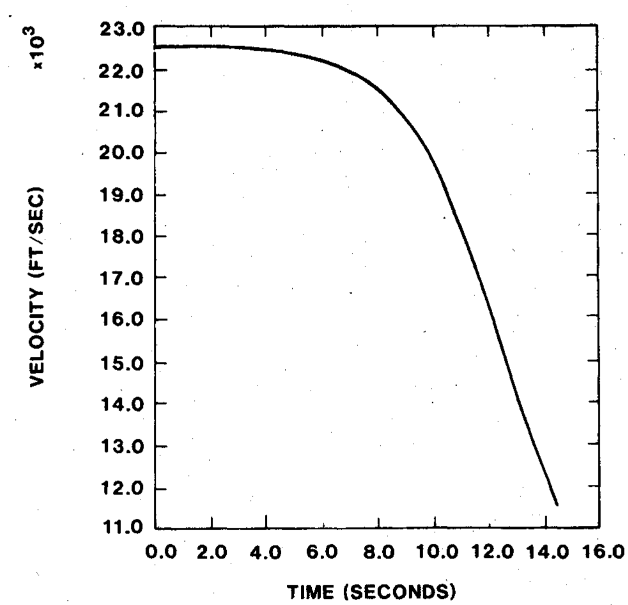

Fig. 8 Simulated re-entry velocity history.

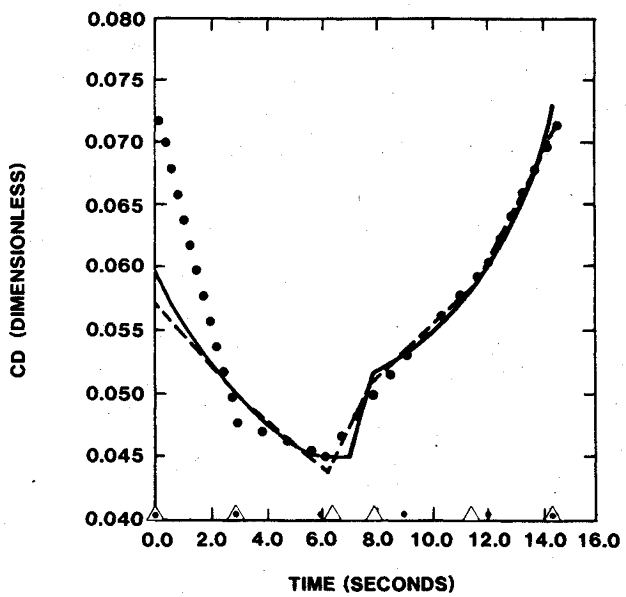

Fig. 9 Comparison of nominal drag history with results for six fixed and four variable nodes.

the function. Difficulties occur with higher order polynomial interpolation in this application when the distance between nodes approaches the distance between observations. In particular, polynomials tend to exhibit oscillatory behavior which can grow in magnitude when little data are available to induce smoothness. However, linear interpolation guarantees that the function takes on values between the values at the adjacent node points. For this reason linear interpolation was used in parameterizing the drag coefficient for the simulated re-entry trajectory to be considered next.

Re-entry Results

The techniques described earlier were used to estimate the drag coefficient and trajectory for a typical re-entry vehicle trajectory. The simulated altitude and velocity histories are shown in Figs. 7 and 8. The drag coefficient history used for the simulation is shown as the solid line in Fig. 9 and is qualitatively representative of actual flight results in which boundary layer transition is known to have occurred. The drag coefficient was parameterized with fixed nodes and estimated along with initial altitude, velocity, and flight-path angle to minimize the weighted sum of squares of residuals in range, azimuth, and elevation. The result of this estimation is shown in Fig. 9 as the dotted line for six uniformly distributed fixed node locations. The fixed locations are indicated by the closed circles on the abscissa.

The nodes were then freed and the resulting drag coefficient history is also shown as the dashed line in Fig. 9. Node locations are indicated by triangles. The results for nodes fixed or free were obtained with identical starting values for the initial vehicle state and drag coefficient history. The weighted square error, J, in range, elevation, and azimuth for this trajectory is a factor of 6 smaller for the variable node approximation than for the fixed node result. In general, improvements on the order of a factor of 2 have been obtained using variable node locations.

Conclusions

A technique has been developed whereby an approximating piecewise continuous function can be parameterized in terms of approximation intervals and the values of the function at the nodes. A variable metric numerical optimization technique has been used to determine the set of these parameters which yields the best approximation of the data in a weighted least-squares manner. The results indicate that better estimates of drag coefficient will be obtained because of the more accurate determination of boundary layer transition time.

Acknowledgments

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References

[1] Williamson, W.E., "The Use of Polynomial Approximations to Calculate Sub-Optimal Controls," *AIAA Journal*, Vol. 9, May 1971, pp. 2271-2273.

[2] Williamson, W.E. Jr., "Minimum and Maximum Endurance Trajectories for Gliding Flight in a Horizontal Plane," *Journal of Guidance and Control*, Vol. 2, Nov.-Dec. 1979, pp. 457-462.

[3] Hull, D.G. and Williamson, W.E., "A Nonlinear Method for Parameter Identification Applied to a Trajectory Estimation Problem," *Journal of Guidance and Control*, Vol. 1, July-Aug. 1978 pp. 286-288.

[4] Akima, H., "A New Method of Interpolation and Smooth Curve Fitting Based on Local Procedures," *Journal of Association for Computing Machinery*, Vol. 17, Oct. 1970, pp. 589-602.

[5] Williamson, W.E., "Square Root Variable Metric Method for Function Minimization," *AIAA Journal*, Vol. 13, Jan. 1975, pp. 107-109.

[6] Pavlidis, T., "Optimal Piecewise Polynomial L_2 Approximation of Functions of One and Two Variables," *IEEE Transactions on Computers*, Jan. 1975, pp. 98-102.

[7] Chang, S.S.L., "Adaptive Curve Fitting and Suboptimization," *IEEE Transactions on Automatic Control*, Dec. 1968, pp. 719-721.

[8] Tomek, I., "Two Algorithms for Piecewise-Linear Continuous Approximation of Functions of One Variable," *IEEE Transactions on Computers*, April 1974, pp. 445-448.